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Planning Production Agenda for Deteriorating Items with Time Exponential-Proportional Demand

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Abstract— A novel approach has been elucidated in this paper for the study of demand using exponential and firms profit with the power of effective inventory control as well as find optimal solution for production with respect to deteriorating items. The objective of this research paper is to develop a mathematical model for deteriorating items such as e.g vegetables, milk, meat, radioactive materials, volatile liquids, etc for proper management of inventory to reduce cost and maximize profit. Managing large amount of inventory is a very difficult task for the firm's. For this studies an inventory model has been illustrated using third order equation. Proper maintaining, dispatching and ordering of these inventories are one of the important decisions of a firm. Hence effective inventory control can substantially contribute in a firm's profit which is studies in this paper. The goal of this research paper most deteriorated item of inventory item, the demand rate has been considered a exponential form. But in a real life these holding cost, deterioration cost vary over time. We develop a deterministic deterioration inventory item with effect in demand rate is exponential.

Keywords— Inventory, Deteriorating, Exponential Demand, Cycle time

I. INTRODUCTION

With the advent of Industrialization, production, planning and its control has become one of the most important strategic decision of an organization on inventory model.[1] and [4] considered deteriorating to be a constant fraction of the on hand inventory while the demand rates is changing linearly. Moreover the cost of material handling plays a significant role in increasing the overall cost of production. To achieve the targets of production at the minimum cost a balance between demand and supply of material inventory is created for optimum utilization of resources. There is a need to effectively monitor and control the cycle of production without affected by the supply of material inventory [7]. The utility has proposed a model by which an optimum solution is derived as a new approach of third order linear equation for those items whose demand changes with time at a constant deterioration rate. [14] discussed the production inventory model for deteriorating items to generate more selling opportunities in multiple market demands. [8] developed an inventory model for decaying items with selling price dependent demand in inflationary environment. An EPQ models was proposed by [12] for deteriorating items with preventive maintenance, random machine breakdown and immediate corrective action. According to [12] corrective and preventive maintenance times are assumed to be stochastic and the unfulfilled demands are lost sales. [11] developed an inventory model for a main class of deteriorating items, under stochastic lead time assumption by considering a non-linear holding cost.

[6] globalization has created many challenges for the business world. The expansion of business involves management of production process which in turn requires proper management and control of inventory. Some items of the inventory are subjected to fast rate of deterioration. Hence there is needed to monitor an effective inventory planning and control.[2] studied three level stock categories for production system boosted by the effective advertisement policy of a firm. This paper provides an optimal solution to control the manufacturing, remanufacturing and disposal rate using sensitivity analysis. Under two level trade credits period [11] studied an inventory model for that inventory which is subjected to continuous deterioration and have a maximum lifetime by using convex fractional programming. Their aim was to obtain an optimum solution at reduced inventory cost. [10] analyses the problem of handling imperfect product quality which is exposed to deterioration at a constant rate. They also examined the different challenges that a retailer faces. This paper aims to develop a model based on advanced preservation technology to maximize profit of the retailer using a sensitivity analysis. The proposed model derives an optimum solution as a new approach for those items whose demand changes with time and have a constant deterioration rate using third order linear equation. Therefore a great deal of effort has been focused on the modeling of the production planning problem in deterministic environment. The assumption of a constant demand rate may not be always appropriate for consumer goods such as milk, meat, vegetables, radioactive materials, volatile liquids, etc. as inventory has a negative

impact on demand due to loss of consumer confidence about the production quality.

Hence in formulating inventory model two factors have been of growing interest, first deterioration of items and second variation in the demand rate with time. Here we are trying to propose inventory model for deteriorating items with exponential demand where a unique optimal cycle time exists to minimize the annual total relevant cost.

The rest of the paper is organized as follows: Section 2 represents the assumptions and notations and section 3 represents problem formulation. Finally, the paper summarizes and concludes in section 4.

II. PROBLEM DEFINITION, ASSUMPTIONS AND NOTATION

2.1 *Assumption:*
The following assumption are used to formulate the problem.
1. The initial inventory level is zero.
2. Lead time is zero.
3. Demand rate is exponential where $D = Ae^{\alpha t}$ where α> 0, t >0 at time t and it is continuous function of time. Here α is a constant.
4. The deteriorating item is constant.
5. The planning horizon is finite.
6. There is no repair or replacement of the deteriorated items.
7. Items are produced / purchased and added to the inventory and the item is a single product; it does not interact with any other inventory items.
8. The production rate is always greater than or equal to the sum of the demand rate

2.2 *NOTATION:*
The following notation are used in our Analysis
1. P-Production rate in units per unit time.
2. Q*-Optimal size of production run.
3. Cp - Production Cost per unit.
4. θ-rate of deterioration.
5. Cd- deterioration cost per unit.
6. C0-SetupCost/ Ordering Cost.
7. Ch-holding cost per unit/year.
8. T-Cycle time.
9. t1-Production time.
10. TC-Total Cost.

III. MATHEMATICAL MODEL

3.1 Development of Mathematical Model
The methodology adopted in this paper to study production planning of deteriorating items with time exponential demand using differential equations. Firstly, for all the periods the differential inventory equations come forth. Next, these differential equation are solved to devise the cost model. The details of this methodology are discussed below. Let us consider a two-stage production-inventory cycle $[0, T]$ of cycle time $T(T>0)$ as shown in Figure 1. It shows inventory level $I(t)$ at time $t(t \geq 0)$ for two stages of the cycle, namely the production stage and the consumption stage. Considering the production time t1 ($0 \leq t \leq t_1$), the production stage covers the period $[0, t_1]$ and the consumption stage covers the period $[t_1, T]$ or $t_1 \leq t \leq T$. This figure represents break up of time when the demand gets reduced

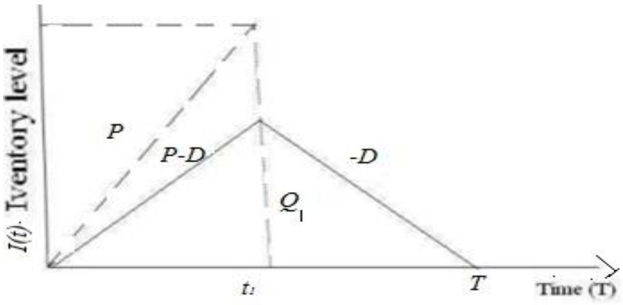

Figure 1: Production-inventory cycle

3.2 *Production period [0, t1]*
The production period $[0, t_1]$ is defined as the time taken for the production of desired unit. During this stage, the inventory of items increases due to production at a rate of P items per unit of time but decreases due to demand at rate D items per unit of time. Here we are interested in exponential demand of $D = Ae^{\alpha t}$, where $\alpha > 0$ is a constant. Note that D is a continuous function of time t. With boundary conditions $I(0) = 0$, $I(T) = 0$, and $I(t_1) = Q_1$, and a deterioration rate of $\theta(0 < \theta < 1)$, the rate at which inventory changes with respect to time over the production period is given by

$$\frac{d^3 I_1(t)}{dt^3} + \theta^3 I_1(t) = P - Ae^{-\theta(t)} \qquad 0 \leq t \leq t1 \ (1)$$

It is also known as inventory differential equation during production period.

3.3 Consumption period $[t_1, T]$
During the consumption period [t1, T], no production occurs. Subsequently deterioration items causes diminution in the inventory level. If such problem arises in a definite time period then this indicated the persistence of seasonal type problems and newspaper inventory type problem. For example, after Christmas the demand of cake and cookies gets reduced. Also a hike in demand of cold drinks is seen in summer season. If there is less demand of items then items start deteriorating as soon as they are produced or after a certain period of time.
Thus, the rate at which inventory changes with respect to time over the consumption period is given by

$$\frac{d^3 I_2(t)}{dt^3} + \theta^3 I_2(t) = -Ae^{-\theta(t)} \qquad 0 \leq t \leq T \ (2)$$

Figure 1 matches to this problem which represents that according to t1 time demand is decreased (We can say that preservative items like Jam, bread starts decaying at that time t1) and T represents complete cycle time. The solutions of differential equations given in (1) and (2) respectively are solution from equation 1st is

$$I_1(t) = c_1 e^{-\theta(t)} + e^{\frac{\theta(t)}{2}}\left(c_2 cos\frac{\sqrt{3}}{2}\theta(t) + c_3 sin\frac{\sqrt{3}}{2}\theta(t)\right) + \frac{P}{\theta^3} - \frac{A}{3\theta^2}e^{-\theta(t)}t_1 \qquad (3)$$

Similarly for equation (2)

$$I_2(t) = c_1 e^{-\theta(t)} + e^{\frac{\theta(t)}{2}}(c_2 cos\frac{\sqrt{3}}{2}\theta(t) + c_3 sin\frac{\sqrt{3}}{2}\theta(t)) - \frac{A}{3\theta^2}e^{-\theta(t)}(T - t_1) \qquad (4)$$

Equating (3) and (4) $I_1(t) = I_2(t)$ at $t = t_1$

$$\frac{P}{\theta^3} - \frac{A}{3\theta^2}e^{-\theta(t_1)}t_1 = - - \frac{A}{3\theta^2}e^{-\theta(t_1)}(T - t_1)$$

Using expansion $e^{-\theta t_1}$ we get

$$\frac{P}{\theta^3} + \frac{A}{3\theta^2}T = \frac{A}{3\theta^2}\theta t_1 T + \frac{2A}{3\theta^2}t_1$$

$$3\theta P + AP = (AT + 2A)t_1$$

$$t_1 = \frac{3\theta P + AT}{AT + 2A}$$

$$t_1 = \frac{\frac{3\theta P}{A} + T}{T + 2} = \frac{R + T}{T + 2}$$

$$t_1 = (R + T)((T + 2)^{-1}$$

$$t_1 = (R + T)(T - 2)$$

We get $t_1 = (R - 2)T - 2R$

3.4 Total Inventory Cost (TIC)

The total inventory cost (TIC) comprises of the ordering cost, holding cost and deteriorating cost, i.e. TIC = Ordering Cost + Holding Cost + Deteriorating Cost.

These costs are evaluated individually as follows:

Ordering Cost per unit time (O_c): Suppose C_o is the total set-up or ordering cost, then the ordering c $[0, T]$ ost per unit time for a cycle over period is given by $O_c = C_o / T$.

Holding Cost/unit per unit time (H_c):

$$HC = \frac{c_h}{T}[\int_0^T I(t)\, dt] \qquad (6)$$

$$HC = \frac{c_h}{T}[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt]$$

Now

$$\int_0^T I(t)dt = \int_0^{t_1} I_1(t)dt + \int_{t_1}^T I_2(t)dt$$

We will get

$$\int_0^T I(t)dt = \frac{AT^2}{2} - \frac{\theta AT^3}{2} - \frac{A^2T^2}{2P} + \frac{\theta A^3 T^3}{2P^2}$$

$$\int_0^T I(t)dt = \int_0^{t_1}(c_1 e^{-\theta(t)} + e^{\frac{\theta(t)}{2}}\left(c_2 cos\frac{\sqrt{3}}{2}\theta(t) + c_3 sin\frac{\sqrt{3}}{2}\theta(t)\right) + \frac{P}{\theta^3} - \frac{A}{3\theta^2}e^{-\theta(t)}t_1)dt + \int_{t_1}^{T}(c_1 e^{-\theta(t)} + e^{\frac{\theta(t)}{2}}(c_2 cos\frac{\sqrt{3}}{2}\theta(t) + c_3 sin\frac{\sqrt{3}}{2}\theta(t)) - \frac{A}{3\theta^2}e^{-\theta(t)}(T - t_1))\, dt$$

$$\int_0^T I_1(t)dt = \frac{P}{\theta^3}t_1 - \left[-\frac{c_1}{\theta} - \frac{c_2}{2\theta} + \frac{\sqrt{3}}{2\theta}c_3 - \frac{At_1}{3\theta^2}\right) + (\frac{-c_1}{\theta}e^{-\theta T} + \frac{c_2}{\theta^2}e^{\frac{\theta}{2}T}\left(\frac{\theta}{2}\, cos\frac{\sqrt{3}}{2}\theta T - \frac{\sqrt{3}}{2}\theta sin\frac{\sqrt{3}}{2}\theta T\right) - \frac{Ae^{-\theta T}}{3\theta^3}t_1 + \frac{c_3}{\theta^2}e^{\frac{\theta}{2}T}\left(\frac{\theta}{2}\, sin\frac{\sqrt{3}}{2}T - \frac{\sqrt{3}}{2}\theta cos\frac{\sqrt{3}}{2}\theta T\right)$$

$$\int_0^T I_1(t)dt = \frac{P}{\theta^3}((R-2)T - 2R) - (-\frac{c_1}{\theta} - \frac{c_2}{2\theta} + \frac{\sqrt{3}}{2\theta}c_3 - \frac{A((R-2)T - 2R)}{3\theta^2} + (\frac{-c_1}{\theta}(1 - \theta T) + \frac{c_2}{\theta^2}\left(1 + \frac{\theta}{2}T\right)\left(\frac{\theta}{2} + \frac{3}{2}\theta^2 T\right) + \frac{A}{3\theta^2}(1 - \theta T)((R-2)T - 2R) + \frac{c_3}{\theta^2}(1 + \frac{\theta}{2}T)(\left(\frac{\sqrt{3}}{4}\theta^2 T - \frac{\sqrt{3}}{2}\theta\right)$$

$$\int_0^T I_1(t)dt = \frac{P}{\theta^3}((R-2)T - \frac{2PR}{\theta^3} + \frac{c_1}{\theta} + \frac{c_2}{2\theta} - \frac{\sqrt{3}}{2\theta}c_3 + \frac{A(R-2)T}{3\theta^2} - \frac{2AR}{3\theta^2} - \frac{c_1}{\theta} + c_1 T + \frac{3}{2}c_2 T + \frac{c_2 T}{4} + \frac{c_2\theta T^2}{4} - \frac{A(R-2)T)}{3\theta^3} + \frac{A2R}{3\theta^2} + \frac{A(R-2)T^2}{3\theta} - \frac{AT2R}{3\theta} + \frac{\sqrt{3}}{4}c_3 T - \frac{\sqrt{3}c_3}{2\theta} + \sqrt{3}\,c_3\frac{\theta}{8}T^2 - c_3\frac{\sqrt{3}}{4}T$$

$$\int_0^T I_1(t)dt = (\frac{P}{\theta^3}(R-2) + c_1 + \frac{3}{2}c_2 + \frac{c_4}{4} - \frac{A}{3\theta}2R + \frac{\sqrt{3}}{4}c_3 - \frac{\sqrt{3}}{4}c_3)^2 T + \left(\frac{c_2\theta^2}{4} + \frac{A}{3\theta} + (R-2) - \frac{\sqrt{3}}{8}c_3\theta\right)T^2 - \frac{2P}{\theta^3}R$$

We assumed

$W=\frac{P}{\theta^3}(R-2) + c_1 + \frac{3}{2}c_2 + \frac{c_4}{4} - \frac{A}{3\theta}2R + \frac{\sqrt{3}}{4}c_3 - \frac{\sqrt{3}}{4}c_3)^2$

$V=\left(\frac{c_2\theta^2}{4} + \frac{A}{3\theta} + (R-2) - \frac{\sqrt{3}}{8}c_3\theta\right)$

$$T_C = \frac{c_0}{T} + \frac{c_h + \theta c_d}{T}(\mathrm{wT} + \mathrm{VT}^2 - \frac{2PR}{\theta^3})$$

$$\frac{dT_C}{dT} = \frac{-c_0}{T^2} + (c_h + \theta c_d)\mathrm{v} + \frac{2PR}{\theta^3 T^2}$$

Putting $\frac{dT_C}{dT} = 0$

$$T = \sqrt{\frac{c_0 - \frac{2PR}{\theta^3}}{c_h + \theta c_d}}$$

$$R = \frac{3P\theta}{A}$$

$$T = \sqrt{\frac{c_0 - \frac{6P^2}{\theta^2 A}}{c_h + \theta c_d}}$$

$\frac{d^2T_C}{dT^2} = \frac{2c_0}{T^3} + \frac{4PR}{\theta^3 T^3}>0$

$\frac{d^2T_C}{dT^2} > 0$

T must be positive gives the minimum value and T gives is optimal time if we run optimal time then we get optimal solution. According to business point of view invest minimum cost and we will get maximum profit. The above standard inventory model gives optimal time. This period to complete one cycle of a task, or to finish off a function, job, or task from beginning to finish it. Cycle time is utilized in differentiating entire duration of a course from its run time.

3.5 *Numerical Example*

1.A Factory buys and use a component for production at Rs. 10 per unit Annual requirement is 2000 units carrying cost of inventory is 10% per annum and ordering cost is Rs. 40 per order The purchase manager propose that as the ordering cost is very high. It is advantage to place a single order for the entire annual requirement. He also says that it we order 2,000 units at a time we can get 2% discount from a supplies. Evaluate the proposal and make your time recommendation.

Solution: Given A=60, 000 gears per year C= 10% of 10=1.D=Aeαt where A=1 and α=1 To find simple solution we take constant equal to 1. D=Aeαt where A=1 and α=1. EOQ is also referred to as the optimum lot size.

EOQ(Q*)=√(2*A*O)/C=400.

Cost of inventory→Purchase Cost+ordering Cost+Carrying cost

No. of order=Annual Demand /EOQ=2000/400=50 Order

Analysis and Results

Table 1: Production inventory model for deteriorating items with exponential Demand

Deteriorati-on rate θ	Demand (in gears per unit/per year)	Cycle Time(T) per unit of product	Production time/ break time t1	Optimal size of production run Q*	Setup Cost per unit/per year	Holding Cost per unit/per year	Deteriorating cost per unit/per year	Total invento-ry cost per year
0.0100	365.10585	17.398745	0.000290	11.548776	229.901642	431.138733	17.964113	679.004517
0.0200	365.050005	8.219807	0.000137	11.547893	486.629456	206.050964	17.170914	709.851318
0.0300	365.031755	5.220623	0.000087	11.547604	766.192078	132.086670	16.510834	914.789551
0.0400	365.02263	3.723144	0.000062	11.547461	1074.360718	95.058304	15.843050	1185.262085
0.0500	365.017155	2.821389	0.000047	11.547373	1417.741333	72.699913	15.145815	1505.587036
0.0600	365.013505	2.216748	0.000037	11.547315	1804.445313	57.656178	14.414044	1876.515503
0.0700	365.01095	1.781920	0.000030	11.547273	2244.769287	46.788715	13.646708	2305.204590
0.0800	365.00876	1.453412	0.000024	11.547241	2752.144287	38.531860	12.843954	2803.520020
0.0900	365.0073	1.195974	0.000020	11.547216	3344.553711	32.016670	12.006252	3388.576660
0.1000	365.00584	0.988449	0.000016	11.547196	4046.742188	26.721893	11.134122	4084.598145

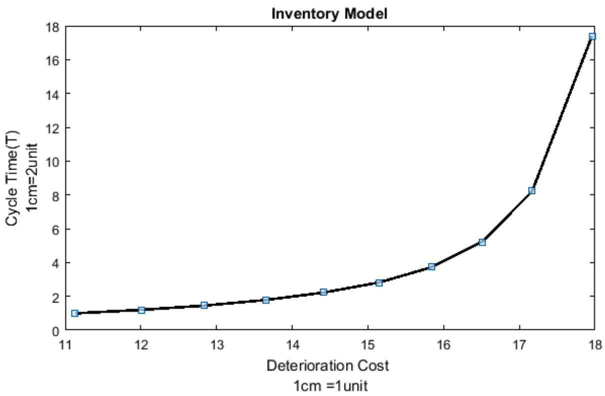

Figure 2: Inventory model, Cycle time vs Deteriorating cost

This graph represents cycle time w.r.t deterioration cost so that one can easily find the deterioration cost within one cycle time. From the observation table 1, a study of rate of deteriorative items with cycle time, optimum quantity setup cost and total cost and it is concluded that when the rate of deteriorative items increases then the setup cost and total cost increases which illustrate apositive relationship between demand, cycle time, deteriorating cost optimum quantity, holding cost and vice versa.

IV. DISCUSSION AND CONCLUSION

In this paper, we have developed inventory model for the deterioration factor takes into consideration as almost all items undergo either direct spoilage (like fruits, vegetables, food stuff etc) or physical decay (in case of radio active substance , volatile liquids vaporization etc.) the deterioration is regular feature in inventory system The main objective of this research paper is to reduce the total time T by considering the exponential demand. The model proposed in this paper can be extended in several ways.

i.This model extend to realistic features such as inventory detailed list of movable items are necessary to manufacture a product. Inventory is the goods and materials.
ii. To calculate the risk factor of the inventory item.
iii. Managing large amount of inventory is a very difficult task for the firm's. Proper maintaining, dispatching and ordering of these inventories is one of the important decision of a firm.
iv. Decision making process will be improved.
v. To Maximize the profit for controlling of inventory items.
vi. Control the deterioration of inventory and economy investment.
vii. Save time and energy & Easy to use.
viii. Platform independent.
ix. Practical Capability Develop.
x. Can be integrated in College Curriculum.

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